B.Tech I Year

Regular Course Handbook

Subject Name: Engineering Mathematics-I (Unit-1)

ENGINEERING MATHEMATICS - I

Subject Code –KAS 103T

SYLLABUS

Unit – 1: (Matrices) [08]

Types of Matrices: Symmetric, Skew-symmetric and Orthogonal Matrices; Complex Matrices, Inverse and Rank of matrix using elementary transformations, Rank-Nullity theorem; System of linear equations, Characteristic equation, Cayley-Hamilton Theorem and its application, Eigen values and eigenvectors; Diagonalisation of a Matrix

Unit – 2: (Differential Calculus- I) [08]

Introduction to limits, continuity and differentiability, Rolle's Theorem, Lagrange's Mean value theorem and Cauchy mean value theorem, Successive Differentiation (nth order derivatives), Leibnitz theorem and its application, Envelope of family of one and two parameter, Curve tracing: Cartesian and Polar co-ordinates

Unit– 3: (Differential Calculus-II) [08]

Partial derivatives, Total derivative, Euler's Theorem for homogeneous functions, Taylor and Maclaurin's theorems for a function of two variables, Maxima and Minima of functions of several variables, Lagrange Method of Multipliers, Jacobians, Approximation of errors

Unit – 4: (Multivariable Calculus-I) [08]

Multiple integration: Double integral, Triple integral, Change of order of integration, Change of variables
Application: Areas and volumes, Center of mass and center of gravity (Constant and variable densities)

Unit – 5 : (Vector Calculus) [08]

Vector identities (without proof), Vector differentiation: Gradient, Curl and Divergence and their Physical interpretation, Directional derivatives. Vector Integration: Line integral, Surface integral, Volume integral, Gauss's Divergence theorem, Green's theorem and Stoke's theorem (without proof) and their applications

B. Tech I Year [Subject Name: ENGINEERING MATHEMATICS – I]

TBL Book Issued to Students

1. A textbook of ENGINEERING MATHEMATICS-I, N. P. Bali, Dr. Manish Goyal.

Reference Books available in Institute Library

1. B. V. Ramana, Higher Engineering Mathematics, McGraw-Hill Publishing Company Ltd., 2008.
2. B. S. Grewal, Higher Engineering Mathematics, Khanna Publisher, 2005.
3. R K. Jain & S R K. Iyenger, Advance Engineering Mathematics, Narosa Publishing House
4. E. Kreyszig, Advance Engineering Mathematics, John Wiley & Sons, 2005.

PREREQUISITES SKILLS

- Basic Knowledge of Mathematics at class XII level.

Course Outcomes: At the end of this course students will demonstrate the ability to:

	Course Outcome (COs)	Blooms Taxonomy Level
CO-1	Remember the concept of matrices and apply for solving linear simultaneous equations.	Remembering, Applying
CO-2	Understand the concept of limit, continuity and differentiability and apply in the study of Rolle's, Lagrange's and Cauchy mean value theorem and Leibnitz theorems.	Understanding, Applying
CO-3	Identify the application of partial differentiation and apply for evaluating maxima, minima, series and Jacobians.	Understanding, Applying
CO-4	Illustrate the working methods of multiple integral and apply for finding area, volume, centre of mass and centre of gravity.	Applying, Evaluating
CO-5	Remember the concept of vector and apply for directional derivatives, tangent and normal planes. Also evaluate line, surface and volume integrals.	Applying, Evaluating

I Year Subjectwise/Unitwise Regular Course Lecture Plan Session 2021-22

Subject Name	Engineering Mathematics-I

Unit No.	Unit Name	Syllabus Topics	Lecture No
1	Matrices	Symmetric, Skew-symmetric, Orthogonal Matrices, Complex Matrices and problems	1
		Inverse of matrix using elementary transformations	2
		Rank of matrix using elementary transformations	3
		Rank of matrix by normal form, Rank-Nullity theorem,	4
		Solution of Non-Homogeneous system of linear equations	5
		Problems of Non-Homogeneous system	6
		Solution of Homogeneous system of linear equations	7
		Eigen values and Eigen vectors	8
		Problems of Eigen values and Eigen vectors,Diagonalisation of a Matrix	9
		Cayley-Hamilton Theorem and its application,	10
2	Differential Calculus-I	Introduction to limits, continuity and Differentiability	11
		Rolle's Theorem, Lagrange's Mean value theorem, Cauchy mean value theorem	12
		Introduction of Successive Differentiation, nth derivative of some elementary functions	13
		Leibnitz's Theorem & nth derivative of product of functions	14
		Relation between yn, yn+1 and yn-2	15
		To find nth derivative of a function at x=0	16
3	Differential Calculus-II	Introduction to partial differentiation and partial derivatives	17
		Chain rule on partial derivatives	18
		Introduction to total differentiation and total derivatives	19
		Euler's Theorem for homogeneous functions	20
		Deductions from Euler's Theorem	21
		Taylor & Maclaurin's theorems for a function of two variables	22
		Maxima and Minima of functions of several variables	23
		Lagrange Method of Multipliers	24
		Problems on Lagrange Method of Multipliers	25
		Introduction to Jacobian	26
		Properties of Jacobian	27
		Jacobian of Implicit Functions	28
		Approximation of errors	29

I Year Subjectwise/Unitwise Regular Course Lecture Plan Session 2021-22

Subject Name	Engineering Mathematics-I

Unit No.	Unit Name	Syllabus Topics	Lecture No
4	Multivariable Calculus-I	Introduction to Double Integral	30
		Problems on Double integral	31
		Double integral in Polar coordinate	32
		Change of order of integration	33
		Area by Double Integral	34
		Introduction of Triple integral,Volume by triple integral	35
		Change of variables in Double and Triple integral	36
		Problems on Change of variables in Double and Triple integral	37
		Center of mass and center of gravity	38
5	Vector Calculus	Gradient	39
		Directional Derivatives	40
		Divergence of a vector and it's physical Interpretations	41
		Curl of a vector and it's physical interpretations &	42
		Line, Surface and Volume Integrals	43
		Applications of Green's Theorem	44
		Applications of Stoke's Theorem	45
		Applications of Gauss Divergence Theorem	46
2	Differential Calculus-I	Envelope of family of one and two parameter	47
		Curve tracing	48

Signature	Ruchi Garg
Name of Subject Head	Dr. Ruchi Garg

B. Tech I Year [Subject Name: Engineering Mathematics]

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		Inverse of matrix using elementary transformations	2
		Rank of matrix using elementary transformations	3
		Rank of matrix by normal form, Rank-Nullity theorem,	4
		Solution of Non-Homogeneous system of linear equations	5
		Problems of Non-Homogeneous system	6
		Solution of Homogeneous system of linear equations	7
		Eigen values and Eigen vectors	8
		Problems of Eigen values and Eigen vectors, Diagonalisation of a Matrix	9
		Cayley-Hamilton Theorem and its application,	10

Matrices (Unit-1)

Def:- An arrangement of no's. in the form of an rectangular array is defined as a matrix.

For eg:-

$$A = \begin{bmatrix} 1 & 2 & 3 & 5 \\ 4 & 2 & 5 & 7 \\ 3 & 4 & 2 & 6 \end{bmatrix}_{3\times4}$$

Unitary Matrix:- A square matrix A is said to be unitary matrix if

$\boxed{A\cdot A^{\theta} = A^{\theta}\cdot A = I}$ where $A^{\theta} = (\bar{A})^T$

Ques1:- If $N = \begin{bmatrix} 0 & 1+2i \\ -1+2i & 0 \end{bmatrix}$ is a matrix, then show that $(I-N)(I+N)^{-1}$ is unitary matrix, where I is the identity matrix. [2012-13]

Soln:- We have, $N = \begin{bmatrix} 0 & 1+2i \\ -1+2i & 0 \end{bmatrix}$

$$I-N = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 1+2i \\ -1+2i & 0 \end{bmatrix} = \begin{bmatrix} 1-0 & -(1+2i) \\ 1-2i & 1-0 \end{bmatrix} \quad (1)$$

$$I+N = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 1+2i \\ -1+2i & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1+2i \\ -1+2i & 1 \end{bmatrix}$$

$|I+N| = 6$

$$Adj(I+N) = \begin{bmatrix} 1 & -1-2i \\ 1-2i & 1 \end{bmatrix} \Rightarrow (I+N)^{-1} = \frac{Adj(I+N)}{|I+N|} = \frac{1}{6}\begin{bmatrix} 1 & -1-2i \\ 1-2i & 1 \end{bmatrix} \quad (2)$$

for unitary matrix, $A\cdot A^{\theta} = I = A^{\theta}\cdot A$

from (1) & (2) we get

B. Tech I Year [Subject Name: Engineering Mathematics]

$$(I-N)(I+N)^{-1} = \frac{1}{6}\begin{bmatrix} 1 & -1-2i \\ 1-2i & 1 \end{bmatrix}\begin{bmatrix} 1 & -1-2i \\ 1-2i & 1 \end{bmatrix} = \frac{1}{6}\begin{bmatrix} -4 & -2-4i \\ 2-4i & -4 \end{bmatrix} = B \text{ (say)}$$

$$\text{Now } (\bar{B})^T = \frac{1}{6}\begin{bmatrix} -4 & 2+4i \\ -2+4i & -4 \end{bmatrix} = B^{\theta}$$

$$\therefore B^{\theta}\cdot B = \frac{1}{36}\begin{bmatrix} -4 & 2+4i \\ -2+4i & -4 \end{bmatrix}\begin{bmatrix} -4 & -2-4i \\ 2-4i & -4 \end{bmatrix} = \frac{1}{36}\begin{bmatrix} 36 & 0 \\ 0 & 36 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \Rightarrow \boxed{B^{\theta}\cdot B = I}$$

Hence Proved.

Ques2:- If $A = \begin{bmatrix} 2 & 3+2i & -4 \\ 3-2i & 5 & 6i \\ -4 & -6i & 3 \end{bmatrix}$ then show that A is Hermetian and iA is skew-Hermetian. [2013-14]

Soln:- We have $A = \begin{bmatrix} 2 & 3+2i & -4 \\ 3-2i & 5 & 6i \\ -4 & -6i & 3 \end{bmatrix}$, $\bar{A} = \begin{bmatrix} 2 & 3-2i & -4 \\ 3+2i & 5 & -6i \\ -4 & 6i & 3 \end{bmatrix}$

$$\text{Now } (\bar{A})^T = \begin{bmatrix} 2 & 3+2i & -4 \\ 3-2i & 5 & 6i \\ -4 & -6i & 3 \end{bmatrix} = A \Rightarrow \boxed{A^{\theta} = A}$$ thus A is Hermetian.

$$\text{Now } iA = i\begin{bmatrix} 2 & 3+2i & -4 \\ 3-2i & 5 & 6i \\ -4 & -6i & 3 \end{bmatrix} \Rightarrow \begin{bmatrix} 2i & 3i-2 & -4i \\ 3i+2 & 5i & -6 \\ -4i & 6 & 3i \end{bmatrix} = B \text{ (say)}$$

$$\bar{B} = \begin{bmatrix} -2i & -3i-2 & 4i \\ -3i+2 & -5i & -6 \\ 4i & 6 & -3i \end{bmatrix}, \quad (\bar{B})^T = \begin{bmatrix} -2i & -3i+2 & 4i \\ -3i-2 & -5i & 6 \\ 4i & -6 & -3i \end{bmatrix}$$

$$= -\begin{bmatrix} 2i & 3i-2 & -4i \\ 3i+2 & 5i & -6 \\ -4i & 6 & 3i \end{bmatrix} = B \Rightarrow \boxed{B = -(\bar{B})^T} \text{ or } \boxed{B = -B^{\theta}}$$

Hence iA is skew-Hermetian.

Ques3:- If $A = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{bmatrix}$ is a unitary matrix, where ω is the complex cube root of unity. [2016-17]

Soln:- We know that, $\omega^2+\omega+1=0$

By quadratic eqn $\omega = -\frac{1}{2} \pm \frac{i\sqrt{3}}{2}$

$\omega^2 = \left(-\frac{1}{2} \pm \frac{i\sqrt{3}}{2}\right)^2 = -\frac{1}{2} - \frac{i\sqrt{3}}{2}$

So, $A = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1 & 1 \\ 1 & -\frac{1}{2}+\frac{i\sqrt{3}}{2} & -\frac{1}{2}-\frac{i\sqrt{3}}{2} \\ 1 & -\frac{1}{2}-\frac{i\sqrt{3}}{2} & -\frac{1}{2}+\frac{i\sqrt{3}}{2} \end{bmatrix}$

$\bar{A} = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1 & 1 \\ 1 & -\frac{1}{2}-\frac{i\sqrt{3}}{2} & -\frac{1}{2}+\frac{i\sqrt{3}}{2} \\ 1 & -\frac{1}{2}+\frac{i\sqrt{3}}{2} & -\frac{1}{2}-\frac{i\sqrt{3}}{2} \end{bmatrix}$

$A^\theta = (\bar{A})^T = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -\frac{1}{2}-\frac{i\sqrt{3}}{2} & -\frac{1}{2}+\frac{i\sqrt{3}}{2} \\ 1 & -\frac{1}{2}+\frac{i\sqrt{3}}{2} & -\frac{1}{2}-\frac{i\sqrt{3}}{2} \end{bmatrix}$

So, $A\cdot A^\theta =$

$\frac{1}{\sqrt{3}}\times\frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1 & 1 \\ 1 & -\frac{1}{2}+\frac{i\sqrt{3}}{2} & -\frac{1}{2}-\frac{i\sqrt{3}}{2} \\ 1 & -\frac{1}{2}-\frac{i\sqrt{3}}{2} & -\frac{1}{2}+\frac{i\sqrt{3}}{2} \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 1 & -\frac{1}{2}-\frac{i\sqrt{3}}{2} & -\frac{1}{2}+\frac{i\sqrt{3}}{2} \\ 1 & -\frac{1}{2}+\frac{i\sqrt{3}}{2} & -\frac{1}{2}-\frac{i\sqrt{3}}{2} \end{bmatrix}$

$= \frac{1}{3}\begin{bmatrix} 3 & 0 & 0 \\ 0 & 1+\frac{1}{4}+\frac{3}{4}+\frac{1}{4}+\frac{3}{4} & 1+\frac{1}{4}-\frac{3}{4}-\frac{i\sqrt{3}}{2}+\frac{1}{4}-\frac{3}{4}+\frac{i\sqrt{3}}{2} \\ 0 & 1+\frac{1}{4}-\frac{3}{4}-\frac{i\sqrt{3}}{2}+\frac{1}{4}-\frac{3}{4}+\frac{i\sqrt{3}}{2} & 1+\frac{1}{4}+\frac{3}{4}+\frac{1}{4}+\frac{3}{4} \end{bmatrix}$

$= \frac{1}{3}\begin{bmatrix} 3 & 0 & 0 \\ 0 & \frac{12}{4} & 1-\frac{1}{2}-\frac{1}{2} \\ 0 & 1-\frac{1}{2}-\frac{1}{2} & \frac{12}{4} \end{bmatrix}$

$= \frac{1}{3}\begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$

Since $\boxed{A\cdot A^\theta = I}$, hence A is a unitary matrix.

Ques4:- Show that the matrix $\begin{bmatrix} \alpha+i\gamma & -\beta+i\delta \\ \beta+i\delta & \alpha-i\gamma \end{bmatrix}$ is unitary if $\alpha^2+\beta^2+\gamma^2+\delta^2=1$. [2017-1…]

Soln:- We have, $A = \begin{bmatrix} \alpha+i\gamma & -\beta+i\delta \\ \beta+i\delta & \alpha-i\gamma \end{bmatrix}$

$A^\theta = \begin{bmatrix} \alpha-i\gamma & \beta-i\delta \\ -\beta-i\delta & \alpha+i\gamma \end{bmatrix}$

for unitary matrix $A\cdot A^\theta = I$

$\Rightarrow \begin{bmatrix} \alpha+i\gamma & -\beta+i\delta \\ \beta+i\delta & \alpha-i\gamma \end{bmatrix}\begin{bmatrix} \alpha-i\gamma & \beta-i\delta \\ -\beta-i\delta & \alpha+i\gamma \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\begin{bmatrix} \alpha^2+\gamma^2+\beta^2+\delta^2 & \alpha\beta-i\alpha\delta+i\beta\gamma+\gamma\delta-\alpha\beta-i\beta\gamma+i\alpha\delta-\delta\gamma \\ \alpha\beta-i\beta\gamma+i\alpha\delta+\gamma\delta-\alpha\beta-i\alpha\delta+i\beta\gamma-\delta\gamma & \beta^2+\delta^2+\alpha^2+\gamma^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} \alpha^2+\beta^2+\gamma^2+\delta^2 & 0 \\ 0 & \alpha^2+\beta^2+\gamma^2+\delta^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow \boxed{\alpha^2+\beta^2+\gamma^2+\delta^2=1}$

Ques5:- Prove that the matrix $\frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1+i \\ 1-i & -1 \end{bmatrix}$ is unitary. [2020-2…]

Soln:- Let $A = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1+i \\ 1-i & -1 \end{bmatrix}$

Now $A^\theta = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1+i \\ 1-i & -1 \end{bmatrix}$

$A\cdot A^\theta = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1+i \\ 1-i & -1 \end{bmatrix}\cdot\frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1+i \\ 1-i & -1 \end{bmatrix}$

$= \frac{1}{\sqrt{3}}\begin{bmatrix} 1+(1+1) & (1+i)-(1+i) \\ (1-i)-1(1-i) & (1+1+1) \end{bmatrix}$

$= \frac{1}{3}\begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$

$\Rightarrow \boxed{A\cdot A^\theta = I}$

thus A is a unitary matrix.

Adjoint of a Square matrix:- The adjoint of a square matrix is the transpose of the matrix obtained by replacing each element of A by its cofactor in |A|.

$$adj\,A = \begin{bmatrix} A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \\ C_1 & C_2 & C_3 \end{bmatrix}$$ where the capital letters denote the co-factors of corresponding small letters in A.

$$|A| = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

Note:- If A is an n-rowed square matrix then $A \cdot (adj\,A) = (adj\,A) \cdot A = |A| I_n$.

Properties of Adjoint:- (1) If A and B are two non-singular square matrices of the same order then

$$\boxed{adj\,(AB) = (adj\,B) \cdot (adj\,A)}$$

(2) If A is a non-singular matrix of order n then,

i) $\boxed{|adj\,A| = |A|^{n-1}}$ ii) $\boxed{adj\,(adj\,A) = |A|^{n-2} \cdot A}$

iii) $\boxed{adj\,A^T = (adj\,A)^T}$

Inverse (or Reciprocal) of a square matrix:- Let A be an n-rowed square matrix. If there exist an n-rowed square matrix B s.t. $\boxed{AB = BA = I}$ then the matrix A is said to be invertible and B is called inverse (or reciprocal) of A.

Note:- 1. To possess inverse, A should be non-singular i.e. $\boxed{|A| \neq 0}$

2. Inverse of A is denoted by A^{-1}, thus $\boxed{B = A^{-1}}$ and $\boxed{AA^{-1} = A^{-1}A = I}$

3. $\boxed{A^{-1} = \frac{adj\,A}{|A|}}$; $\boxed{|A| \neq 0}$

4. $\boxed{|A^{-1}| = |A|^{-1}}$

Ques 6:- If $A = \begin{bmatrix} 1 & 2 & 1 \\ a & 0 & 4 \\ 1 & 1 & 1 \end{bmatrix}$ and $adj(adj\,A) = A$, find a. [2011-12]

Soln:- Let $A = \begin{bmatrix} 1 & 2 & 1 \\ a & 0 & 4 \\ 1 & 1 & 1 \end{bmatrix}$, $adj\,A = \begin{bmatrix} -4 & -(a-4) & a \\ -1 & 0 & 1 \\ 8 & -(4-a) & -2a \end{bmatrix}^T = \begin{bmatrix} -4 & -1 & 8 \\ 4-a & 0 & a-4 \\ a & 1 & -2a \end{bmatrix} = B$ (say)

$$adj\,B = adj\,(adj\,A) = \begin{bmatrix} 4-a & 8-2a & 4-a \\ 4a-a^2 & 0 & 16-4a \\ 4-a & 4-a & 4-a \end{bmatrix} = A \text{ (given)}$$

$$\Rightarrow (4-a)\begin{bmatrix} 1 & 2 & 1 \\ a & 0 & 4 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 1 \\ a & 0 & 4 \\ 1 & 1 & 1 \end{bmatrix} \Rightarrow 4-a=1 \Rightarrow \boxed{a=3}$$

Ques 7:- Explain the working rule to find the inverse of a matrix A by elementary row and column transformations. [2012-13]

Soln:- Write $\boxed{A = IA}$

Now try to convert matrix A on LHS to the identity matrix by applying elementary row operations.

The elementary row transformations which reduces a square matrix A to the unit matrix (Identity matrix), when applied to the unit matrix (Identity matrix) gives the inverse matrix A^{-1}.

Ques 8:- For the given matrix $A = \begin{bmatrix} -5 & -3 \\ 2 & 0 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ prove that $A^3 = 19A + 30I$. [2016-17]

Soln:- LHS We have, $A = \begin{bmatrix} -5 & -3 \\ 2 & 0 \end{bmatrix} \Rightarrow A^2 = A \times A = \begin{bmatrix} -5 & -3 \\ 2 & 0 \end{bmatrix}\begin{bmatrix} -5 & -3 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 19 & 15 \\ -10 & -6 \end{bmatrix}$

$$A^3 = A^2 \times A = \begin{bmatrix} 19 & 15 \\ -10 & -6 \end{bmatrix} \times \begin{bmatrix} -5 & -3 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} -65 & -57 \\ 38 & 30 \end{bmatrix}$$

RHS: $19A + 30I = 19\begin{bmatrix} -5 & -3 \\ 2 & 0 \end{bmatrix} + 30\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -95 & -57 \\ 38 & 0 \end{bmatrix} + \begin{bmatrix} 30 & 0 \\ 0 & 30 \end{bmatrix}$

$$= \begin{bmatrix} -65 & -57 \\ 38 & 30 \end{bmatrix} = A^3 \text{ (LHS)}$$ Hence proved.

Ques 9:- Compute the inverse of the matrix $\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$ by employing elementary row transformations. [2017-18]

Soln:- Let $A \sim I$

$$\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Operating R_{12}

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 3 & 1 & 1 \end{bmatrix} \sim \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Operating $R_{31}(-3)$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & -5 & -8 \end{bmatrix} \sim \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & -3 & 1 \end{bmatrix}$$

Operating $R_{12}(-2)$, $R_{32}(5)$

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{bmatrix} \sim \begin{bmatrix} -2 & 1 & 0 \\ 1 & 0 & 0 \\ 5 & -3 & 1 \end{bmatrix}$$

Operating $R_{13}(\frac{1}{2})$, $R_{23}(-1)$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix} \sim \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ -4 & 3 & -1 \\ 5 & -3 & 1 \end{bmatrix}$$

Operating $R_3(\frac{1}{2})$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \sim \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ -4 & 3 & -1 \\ 5/2 & -3/2 & 1/2 \end{bmatrix}$$

$\boxed{I \sim A^{-1}}$

$$\boxed{A^{-1} = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ -4 & 3 & -1 \\ 5/2 & -3/2 & 1/2 \end{bmatrix}}$$

Ques 10:- Find the inverse employing elementary transformations:- [2018-19]

$$A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$$

Soln:- $A = IA$

$$\Rightarrow \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

$$\begin{bmatrix} 1 & -1 & \frac{4}{3} \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \quad (R_1 \to \frac{R_1}{3})$$

$$\begin{bmatrix} 1 & -1 & \frac{4}{3} \\ 0 & -1 & 4/3 \\ 0 & -1 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & 0 & 0 \\ -\frac{2}{3} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \quad (R_2 \to R_2 - 2R_1)$$

$$\begin{bmatrix} 1 & -1 & 4/3 \\ 0 & 1 & -4/3 \\ 0 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 1/3 & 0 & 0 \\ 2/3 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \quad (R_2 \to -R_2)$$

$$\begin{bmatrix} 1 & -1 & 4/3 \\ 0 & 1 & -4/3 \\ 0 & 0 & -1/3 \end{bmatrix} = \begin{bmatrix} 1/3 & 0 & 0 \\ 2/3 & -1 & 0 \\ 2/3 & -1 & 1 \end{bmatrix} A \quad (R_3 \to R_3 + R_2)$$

$$\begin{bmatrix} 1 & -1 & 4/3 \\ 0 & 1 & -4/3 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1/3 & 0 & 0 \\ 2/3 & -1 & 0 \\ -2 & 3 & -3 \end{bmatrix} A \quad (R_3 \to -3R_3)$$

$$\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -4 & 4 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix} A$$

$R_1 \to R_1 - \frac{4}{3} R_3$
$R_2 \to R_2 + \frac{4}{3} R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix} A \quad (R_1 \to R_1 + R_2)$$

$$\Rightarrow \boxed{A^{-1} = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix}}$$

Ques 11:- Find the inverse of the matrix $A = \begin{bmatrix} 2 & 3 & 4 \\ 4 & 3 & 1 \\ 1 & 2 & 4 \end{bmatrix}$ [2020-21]

Soln:- Write $A = IA$

$$\begin{bmatrix} 2 & 3 & 4 \\ 4 & 3 & 1 \\ 1 & 2 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

$$\begin{bmatrix} 2 & 3 & 4 \\ 0 & -3 & -7 \\ 1 & 2 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \quad (R_2 \to R_2 - 2R_1)$$

$$\begin{bmatrix} 1 & 3/2 & 2 \\ 0 & -3 & -7 \\ 1 & 2 & 4 \end{bmatrix} = \begin{bmatrix} 1/2 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \quad (R_1 \to R_1/2)$$

$$\begin{bmatrix} 1 & 3/2 & 2 \\ 0 & -3 & -7 \\ 0 & 1/2 & 2 \end{bmatrix} = \begin{bmatrix} 1/2 & 0 & 0 \\ -2 & 1 & 0 \\ -1/2 & 0 & 1 \end{bmatrix} A \quad (R_3 \to R_3 - R_1)$$

$$\begin{bmatrix} 1 & 3/2 & 2 \\ 0 & 1 & 7/2 \\ 0 & 1/2 & 2 \end{bmatrix} = \begin{bmatrix} 1/2 & 0 & 0 \\ 2/3 & -1/3 & 0 \\ -1/2 & 0 & 1 \end{bmatrix} A \quad (R_2 \to -\frac{1}{3} R_2)$$

$$\begin{bmatrix} 1 & 0 & -3/2 \\ 0 & 1 & 7/3 \\ 0 & 0 & 5/6 \end{bmatrix} = \begin{bmatrix} -1/2 & 1/2 & 0 \\ 2/3 & -1/3 & 0 \\ -5/6 & 1/6 & 1 \end{bmatrix} A$$

$(R_3 \to R_3 - \frac{1}{2} R_2)$

$$\begin{bmatrix} 1 & 0 & -3/2 \\ 0 & 1 & 7/3 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -1/2 & 1/2 & 0 \\ 2/3 & -1/3 & 0 \\ -1 & 1/5 & 6/5 \end{bmatrix} A \quad (R_3 \to \frac{6}{5} R_3)$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 4/5 & 9/5 \\ 3 & -4/5 & -14/5 \\ -1 & 1/5 & 6/5 \end{bmatrix} A$$

$R_1 \to R_1 + \frac{3}{2} R_3$
$R_2 \to R_2 - \frac{7}{3} R_3$

$$\boxed{A^{-1} = \begin{bmatrix} -2 & 4/5 & 9/5 \\ 3 & -4/5 & -14/5 \\ -1 & 1/6 & 6/5 \end{bmatrix}}$$

Rank of a Matrix :-

The rank of a matrix is said to be r if:-

a) It has atleast one non-zero minor of order r.

b) Every minor of A of order higher than r is zero.

Symbolically, rank of A = r is written as $\boxed{\rho(A) = r}$

Note:- 1. If A is a null-matrix, then $\boxed{\rho(A) = 0}$.

2. If A is not a null-matrix, then $\boxed{\rho(A) \geq 1}$.

3. If A is a non-singular $n \times n$ matrix, then $\boxed{\rho(A) = n}$.

4. If I_n is the $n \times n$ unit matrix, then $|I_n| = 1 \neq 0 \Rightarrow \boxed{\rho(I_n) = n}$.

5. If A is an $m \times n$ matrix, then $\boxed{\rho(A) \leq \text{min. of } m \text{ and } n}$

6. If all minors of order r are equal to zero, then $\boxed{\rho(A) < r}$

Methods of finding Rank:-

(1) By triangular form or Echelon form.

$\boxed{\text{Rank} = \text{No. of non-zero rows in upper triangular form}}$

Note:- Non-zero row is that row which does not contain all the elements as zero.

(2) Normal form or Cannonical form.

By performing elementary transformations, any non-zero matrix A can be reduced to one of the following four forms called the normal form of A:-

i) I_r ii) $[I_r \; 0]$ iii) $\begin{bmatrix} I_r \\ 0 \end{bmatrix}$ iv) $\begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}$

The no. r so obtained is called the rank of A and we write $\boxed{\rho(A) = r}$

Ques12:- Find the value of P for which the matrix $A = \begin{bmatrix} 3 & P & P \\ P & 3 & P \\ P & P & 3 \end{bmatrix}$ is be of rank 1. [2011-12]

Soln:- Since the rank of the matrix A is given as 1, thus by def. of rank all two order minors of A should be zero

$\Rightarrow \begin{vmatrix} 3 & P \\ P & 3 \end{vmatrix} = 0 \Rightarrow 9 - P^2 = 0 \Rightarrow \boxed{P = 3}$, neglecting -3 ∵ it does not satisfy the condition of rank 1.

Ques13:- Reduce A to the echelon form and then to its row cannonical form where $A = \begin{bmatrix} 1 & 3 & -1 & 2 \\ 0 & 11 & -5 & 3 \\ 2 & -5 & 3 & 1 \\ 4 & 1 & 1 & 5 \end{bmatrix}$. Hence find the rank of A. [2014-15]

Soln:- $A = \begin{bmatrix} 1 & 3 & -1 & 2 \\ 0 & 11 & -5 & 3 \\ 2 & -5 & 3 & 1 \\ 4 & 1 & 1 & 5 \end{bmatrix}$

$\sim \begin{bmatrix} 1 & 3 & -1 & 2 \\ 0 & 11 & -5 & 3 \\ 0 & -11 & 5 & -3 \\ 0 & -11 & 5 & -3 \end{bmatrix}$ $R_3 \to R_3 - 2R_1$, $R_4 \to R_4 - 4R_1$

$\sim \begin{bmatrix} 1 & 3 & -1 & 2 \\ 0 & 11 & -5 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ $R_3 \to R_3 + R_2$, $R_4 \to R_4 + R_2$

$\sim \begin{bmatrix} 1 & 3 & -1 & 2 \\ 0 & 1 & -5/11 & 3/11 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ $R_2 \to \frac{1}{11} R_2$

Rank of A = No. of non-zero rows

$\boxed{\rho(A) = 2}$

Ques14:- Using elementary transformations, find the rank of the following matrix:- $A = \begin{bmatrix} -2 & -1 & 3 & -1 \\ 1 & 2 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$ [2017-18]

Soln:- Since $A = \begin{bmatrix} -2 & -1 & 3 & -1 \\ 1 & 2 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 2 & -3 & -1 \\ -2 & -1 & 3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$ $R_1 \leftrightarrow R_2$

$$= \begin{bmatrix} 1 & 2 & -3 & -1 \\ 0 & 3 & -3 & -3 \\ 0 & -2 & 4 & 2 \\ 0 & 1 & 1 & -1 \end{bmatrix} \begin{matrix} R_2 \to R_2 + 2R_1 \\ R_3 \to R_3 - R_1 \end{matrix}$$

$$= \begin{bmatrix} 1 & 2 & -3 & -1 \\ 0 & 3 & -3 & -3 \\ 0 & -2 & 4 & 2 \\ 0 & 0 & 2 & 0 \end{bmatrix} R_4 \to R_4 - \frac{1}{3}R_2$$

$$= \begin{bmatrix} 1 & 2 & -3 & -1 \\ 0 & 3 & -3 & -3 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 2 & 0 \end{bmatrix} R_3 \to R_3 + \frac{2}{3}R_2$$

$$= \begin{bmatrix} 1 & 2 & -3 & -1 \\ 0 & 3 & -3 & -3 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} R_4 \to R_4 - R_3$$

$$= \begin{bmatrix} 1 & 2 & -3 & -1 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} R_2 \to \frac{1}{3}R_2 \\ R_3 \to \frac{1}{2}R_3 \end{matrix}$$

No. of non-zero rows = 3

$\boxed{\rho(A) = 3}$

Ques 15:- Find the rank of the matrix:- $\begin{bmatrix} 2 & 2 & 2 \\ 2 & 2 & 2 \\ 2 & 2 & 2 \end{bmatrix}$ [2018-19]

Soln:- Let $A = \begin{bmatrix} 2 & 2 & 2 \\ 2 & 2 & 2 \\ 2 & 2 & 2 \end{bmatrix}$

$$\sim \begin{bmatrix} 2 & 2 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{matrix} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \end{matrix}$$

No. of non-zero rows = 1

$\boxed{\rho(A) = 1}$

Ques 16:- Find the value of 'b' so that rank of $A = \begin{bmatrix} 2 & 4 & 2 \\ 3 & 1 & 2 \\ 1 & 0 & b \end{bmatrix}$ is 2. [2019-20]

Soln:- Since $\rho(A) = 2 \Rightarrow |A| = 0$

$$\begin{vmatrix} 2 & 4 & 2 \\ 3 & 1 & 2 \\ 1 & 0 & b \end{vmatrix} = 0$$

$$2(b-0) - 4(3b-2) + 2(0-1) = 0$$

$$\Rightarrow 2b - 12b + 8 - 2 = 0$$

$$\Rightarrow \boxed{b = 3/5}$$

Ques 17:- Determine the rank of the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$ [2013-14]

Soln:- Let $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$

$$\sim \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & -1 & 0 \end{bmatrix} R_3 \to R_3 - R_1$$

$$\sim \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} R_3 \to R_3 + R_2$$

$\Rightarrow$ No. of non-zero rows = 3

$\boxed{\rho(A) = 3}$

Ques 18:- Reduce the matrix $\begin{bmatrix} 1 & 1 & 1 \\ 3 & 1 & 1 \end{bmatrix}$ in to the normal form and find its rank. [2017-18]

Soln:- Let $A = \begin{bmatrix} 1 & 1 & 1 \\ 3 & 1 & 1 \end{bmatrix}$

$$\sim \begin{bmatrix} 1 & 1 & 1 \\ 0 & -2 & -2 \end{bmatrix} R_2 \to R_2 - 3R_1$$

$$\sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & -2 & -2 \end{bmatrix} \begin{matrix} C_2 \to C_2 - 2C_1 \\ C_3 \to C_3 - 2C_1 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -2 \end{bmatrix} C_2 \to -\frac{1}{2}C_2$$

$$\sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} C_3 \to C_3 + 2C_2$$

$$\sim \begin{bmatrix} I_2 & 0 \end{bmatrix}$$

$\boxed{\rho(A) = 2}$

Ques 19:- Reduce the matrix A to its normal form when $A = \begin{bmatrix} 1 & 2 & -1 & 4 \\ 2 & 4 & 3 & 4 \\ 1 & 2 & 3 & 4 \\ -1 & -2 & 6 & -7 \end{bmatrix}$. Hence find the rank of A. [2018-19]

Soln:- $A = \begin{bmatrix} 1 & 2 & -1 & 4 \\ 2 & 4 & 3 & 4 \\ 1 & 2 & 3 & 4 \\ -1 & -2 & 6 & -7 \end{bmatrix}$

$$\sim \begin{bmatrix} 1 & 2 & -1 & 4 \\ 0 & 0 & 5 & -4 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & 5 & -3 \end{bmatrix} \begin{matrix} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - R_1 \\ R_4 \to R_4 + R_1 \end{matrix}$$

$$\sim\begin{bmatrix}1&0&0&0\\0&0&5&-4\\0&0&4&0\\0&0&5&-3\end{bmatrix}\quad \begin{matrix}C_2\to C_2-2C_1\\C_3\to C_3+C_1\\C_4\to C_4-4C_1\end{matrix}$$

$$\sim\begin{bmatrix}1&0&0&0\\0&5&0&-4\\0&4&0&0\\0&5&0&-3\end{bmatrix}\quad C_3\leftrightarrow C_2$$

$$\sim\begin{bmatrix}1&0&0&0\\0&5&0&-4\\0&0&0&16/5\\0&0&0&1\end{bmatrix}\quad \begin{matrix}R_3\to R_3-\frac{4}{5}R_2\\R_4\to R_4-R_2\end{matrix}$$

$$\sim\begin{bmatrix}1&0&0&0\\0&5&-4&0\\0&0&16/5&0\\0&0&1&0\end{bmatrix}\quad C_4\leftrightarrow C_3$$

$$\sim\begin{bmatrix}1&0&0&0\\0&5&0&0\\0&0&16/5&0\\0&0&0&0\end{bmatrix}\quad \begin{matrix}R_2\to R_2+\frac{5}{4}R_3\\R_4\to R_4-\frac{5}{16}R_3\end{matrix}$$

$$\sim\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&0\end{bmatrix}\quad \begin{matrix}R_2\to\frac{1}{5}R_2\\R_3\to\frac{5}{16}R_3\end{matrix}$$

$$\sim\begin{bmatrix}I_3&0\\0&0\end{bmatrix}\Rightarrow \boxed{\rho(A)=3}$$

Ques 20:- Find the rank of $A=\begin{bmatrix}1&3&4&2\\2&-1&3&2\\3&-5&2&2\\6&-3&8&6\end{bmatrix}$ by normal form. [2019-20]

Soln:- $A=\begin{bmatrix}1&3&4&2\\2&-1&3&2\\3&-5&2&2\\6&-3&8&6\end{bmatrix}$

$$\sim\begin{bmatrix}1&3&4&2\\0&-7&-5&-2\\0&-14&-10&-4\\0&-21&-16&-6\end{bmatrix}\quad \begin{matrix}R_2\to R_2-2R_1\\R_3\to R_3-3R_1\\R_4\to R_4-6R_1\end{matrix}$$

$$\sim\begin{bmatrix}1&0&0&0\\0&-7&-5&-2\\0&-14&-10&-4\\0&-21&-16&-6\end{bmatrix}\quad \begin{matrix}C_2\to C_2-3C_1\\C_3\to C_3-4C_1\\C_4\to C_4-2C_1\end{matrix}$$

$$\sim\begin{bmatrix}1&0&0&0\\0&-7&-5&-2\\0&0&0&0\\0&0&-1&0\end{bmatrix}\quad \begin{matrix}R_3\to R_3-2R_2\\R_4\to R_4-3R_2\end{matrix}$$

$$\sim\begin{bmatrix}1&0&0&0\\0&-7&-5&-2\\0&0&-1&0\\0&0&0&0\end{bmatrix}\quad R_3\leftrightarrow R_4$$

$$\sim\begin{bmatrix}1&0&0&0\\0&-7&0&0\\0&0&-1&0\\0&0&0&0\end{bmatrix}\quad \begin{matrix}C_3\to C_3-\frac{5}{7}C_2\\C_4\to C_4-\frac{2}{7}C_2\end{matrix}$$

$$\sim\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&0\end{bmatrix}\quad \begin{matrix}R_2\to-\frac{1}{7}R_2\\R_3\to-R_3\end{matrix}$$

$$\sim\begin{bmatrix}I_3&0\\0&0\end{bmatrix}$$ which is a normal form.

$$\Rightarrow \boxed{\rho(A)=3}$$

Ques 21:- State Rank-Nullity th. [2020-21]

Soln:- **Statement:-** If A is an $m\times n$ matrix over some field then

$$\boxed{\text{Rank}(A)+\text{Nullity}(A)=n}$$

This applies to linear maps also. Let V be a finite dimensional vector space and W be a vector space over some field. Let $T: V\to W$ be a linear map, then

$$\boxed{\text{Rank}(T)+\text{Nullity}(T)=\dim(V)}$$

Ques 22:- For non-singular matrices P and Q s.t. PAQ is normal form $\begin{bmatrix}1&1&2\\1&2&3\\0&1&1\end{bmatrix}$. [2020-21]

Soln:- Let $A\sim I_3AI_3$

$$\Rightarrow\begin{bmatrix}1&1&2\\1&2&3\\0&1&1\end{bmatrix}\sim\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}A\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

$$\begin{bmatrix}1&1&2\\0&1&1\\0&1&1\end{bmatrix}\sim\begin{bmatrix}1&0&0\\-1&1&0\\0&0&1\end{bmatrix}A\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}\quad R_2\to R_2-R_1$$

$$\begin{bmatrix}1&1&2\\0&0&1\\0&1&1\end{bmatrix}\sim\begin{bmatrix}2&-1&0\\-1&1&0\\0&0&1\end{bmatrix}A\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}\quad R_1\to R_1-R_2$$

$$\begin{bmatrix}1&1&2\\0&1&1\\0&0&1\end{bmatrix}\sim\begin{bmatrix}2&-1&0\\0&0&1\\-1&1&1\end{bmatrix}A\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}\quad R_2\leftrightarrow R_3$$

$$\begin{bmatrix}1&1&1\\0&1&0\\0&0&1\end{bmatrix}\sim\begin{bmatrix}2&-1&0\\0&0&1\\-1&1&0\end{bmatrix}A\begin{bmatrix}1&0&0\\0&1&-1\\0&0&1\end{bmatrix}\quad C_3\to C_3-C_2$$

$$\begin{bmatrix}1&0&1\\0&1&0\\0&0&1\end{bmatrix}\sim\begin{bmatrix}2&-1&-1\\0&0&1\\-1&1&0\end{bmatrix}A\begin{bmatrix}1&0&0\\0&1&-1\\0&0&1\end{bmatrix}\quad R_1\to R_1-R_2$$

$$\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}\sim\underbrace{\begin{bmatrix}2&-1&-1\\0&0&1\\-1&1&0\end{bmatrix}}_{P}A\underbrace{\begin{bmatrix}1&0&-1\\0&1&-1\\0&0&1\end{bmatrix}}_{Q}\quad C_3\to C_3-C_1$$

$$\boxed{N=PAQ}$$

Linear Dependence and Independence of vectors:-

Vectors (matrices) $X_1, X_2, \ldots X_n$ are said to be dependent if

1) all the vectors (row or column matrices) are of the same order.
2) n scalars $\lambda_1, \lambda_2, \ldots \lambda_n$ (not all zero) exist s.t.

$$\boxed{\lambda_1 X_1 + \lambda_2 X_2 + \lambda_3 X_3 + \cdots + \lambda_n X_n = 0}$$

Otherwise they are linearly independent.

Note:- Linearly Dependence and Independence by Rank Method

1: If the rank of the matrix of the given vectors is equal to the no. of vectors, then the vectors are L.I.

2: If the rank of the matrix of the given vectors is less than the no. of vectors then the vectors are L.D.

Ques 23:- Examine whether the vectors $X_1 = [3, 1, 1]$, $X_2 = [2, 0, -1]$ and $X_3 = [4, 2, 1]$ are linearly independent. [2015-16]

Soln:- Let A be the matrix formed by the vectors X_1, X_2 and X_3. i.e. $\begin{bmatrix} 3 & 1 & 1 \\ 2 & 0 & -1 \\ 4 & 2 & 1 \end{bmatrix}$

$$\sim A = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 0 & -1 \\ 4 & 2 & 1 \end{bmatrix} R_1 \to R_1 - R_2 \sim \begin{bmatrix} 1 & 1 & 2 \\ 0 & -2 & -5 \\ 0 & -2 & -7 \end{bmatrix} \begin{matrix} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - 4R_1 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 1 & 2 \\ 0 & -2 & -5 \\ 0 & 0 & -2 \end{bmatrix} R_3 \to R_3 - R_2 \Rightarrow \boxed{\rho(A) = 3 = \text{no. of non-zero rows.}}$$

and $\boxed{\rho(A) = \text{no. of variables}}$

Thus, the given set of vectors are linearly independent.

Ques 24:- Show that the vectors (1, 6, 4), (0, 2, 3) and (0, 1, 2) are linearly independent. [2019-20]

Soln:- Let $X_1 = (1, 6, 4)$, $X_2 = (0, 2, 3)$ and $X_3 = (0, 1, 2)$

$$A = \begin{bmatrix} 1 & 6 & 4 \\ 0 & 2 & 3 \\ 0 & 1 & 2 \end{bmatrix} \sim \begin{bmatrix} 1 & 6 & 4 \\ 0 & 2 & 3 \\ 0 & 0 & 1/2 \end{bmatrix} R_3 \to R_3 - \tfrac{1}{2} R_2$$

$\sim \boxed{\rho(A) = 3 = \text{no. of non-zero rows} = \text{no. of variables}}$

Thus, the given set of vectors are linearly independent.

Ques 25:- System of Non-Homogeneous Linear Equations

A system of non-Homogeneous linear eq's $AX = B$

Find R(A) and R(C)

If R(A) = R(C) → Solution exists, sys. is consistent

- R(A) = R(C) = n (no. of unknowns) → Unique solution
- R(A) = R(C) < n (no. of unknowns) → Infinite no. of solutions

If R(A) ≠ R(C) → No solution, system is inconsistent

Ques25:- Show that the system of eq^{n}s $3x+4y+5z=A$, $4x+5y+6z=B$, $5x+6y+7z=C$ are consistent only if A, B and C are in arithmetic progression (AP). [2011-12]

Soln:- Let $AX=B$, be the given sys. of eq^{n}s

$$\Rightarrow C=[A:B]=\left[\begin{array}{ccc|c}3&4&5&A\\4&5&6&B\\5&6&7&C\end{array}\right]$$

$$\sim\left[\begin{array}{ccc|c}3&4&5&A\\1&1&1&B-A\\2&2&2&C-A\end{array}\right]\begin{array}{l}R_2\to R_2-R_1\\R_3\to R_3-R_1\end{array}$$

$$\sim\left[\begin{array}{ccc|c}1&1&1&B-A\\3&4&5&A\\2&2&2&C-A\end{array}\right]R_1\leftrightarrow R_2$$

$$\sim\left[\begin{array}{ccc|c}1&1&1&B-A\\0&1&2&4A-3B\\0&0&0&C-2B+A\end{array}\right]\begin{array}{l}R_2\to R_2-3R_1\\R_3\to R_3-2R_1\end{array}$$

which is echelon form

for consistency $\rho(A)=\rho[A|B]$

i.e. we must have $C-2B+A=0$

$\Rightarrow C+A=2B$

$\Rightarrow \boxed{B=\frac{C+A}{2}}$

i.e. A, B and C are in A.P.

Hence Proved.

Ques26:- Investigate for what values of λ and μ the simultaneous eq^{n}s: $x+y+z=6$, $x+2y+3z=10$ and $x+2y+\lambda z=\mu$ have i) No soln. ii) a unique soln. and iii) infinite no. of solns. [2012-13] [2015-16]

Soln:- Augmented matrix $[A:B]=\left[\begin{array}{ccc|c}1&1&1&6\\1&2&3&10\\1&2&\lambda&\mu\end{array}\right]$

Operating $R_{21}(-1), R_{31}(-1)$

$$\sim\left[\begin{array}{ccc|c}1&1&1&6\\0&1&2&4\\0&1&\lambda-1&\mu-6\end{array}\right]$$

$$\sim R_{32}(-1)\left[\begin{array}{ccc|c}1&1&1&6\\0&1&2&4\\0&0&\lambda-3&\mu-10\end{array}\right]$$

Case I:- If $\lambda=3, \mu\neq10 \Rightarrow \rho(A)=2$, $\rho[A:B]=3$

$\Rightarrow \rho(A)\neq\rho[A:B]$ $\therefore$ sys. has no solution.

Case II:- If $\lambda\neq3$, μ may have any value

$\rho(A)=\rho[A:B]=3=$ no. of unknowns.

$\therefore$ sys. has unique soln.

Case III:- If $\lambda=3, \mu=10 \Rightarrow \rho(A)=\rho[A:B]=2<$ no. of unknowns

$\therefore$ sys. has infinite no. of solns.

Ques27:- Test the consistency and solve the following sys. of eq^{n}s: $2x-y+3z=8$, $-x+2y+z=4$ and $3x+y-4z=0$ [2013-14]

Soln:- Augmented matrix $C=[A:B]=\left[\begin{array}{ccc|c}2&-1&3&8\\-1&2&1&4\\3&1&-4&0\end{array}\right]$

$$\sim\left[\begin{array}{ccc|c}-1&2&1&4\\2&-1&3&8\\3&1&-4&0\end{array}\right]R_1\leftrightarrow R_2$$

$$\sim\left[\begin{array}{ccc|c}-1&2&1&4\\0&3&5&16\\0&7&-1&12\end{array}\right]\begin{array}{l}R_2\to R_2+2R_1\\R_3\to R_3+3R_1\end{array}$$

$$\sim\left[\begin{array}{ccc|c}-1&2&1&4\\0&21&35&112\\0&21&-3&36\end{array}\right]\begin{array}{l}R_2\to 7R_2\\R_3\to 3R_3\end{array}\sim\left[\begin{array}{ccc|c}-1&2&1&4\\0&21&35&112\\0&0&-38&-76\end{array}\right]R_3\to R_3-R_2$$

which is an echelon form. $\rho(A) = 3$, $\rho[A:B] = 3$ = no. of variables
$\Rightarrow$ system is consistent and have unique soln.

$AX = B$

$$\begin{bmatrix} -1 & 2 & 1 \\ 0 & 21 & 35 \\ 0 & 0 & -38 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 112 \\ -76 \end{bmatrix} \Rightarrow \begin{aligned} -x+2y+z &= 4 \quad (1) \\ 21y+35z &= 112 \quad (2) \\ -38z &= -76 \quad (3) \end{aligned}$$

from (3) $\boxed{z=2}$, from (2) $\boxed{y=2}$ and from (1) $\boxed{x=2}$ is the unique soln. of above sys. of eqns.

Ques 28:- Solve by calculating the inverse by elementary row operations: $x_1+x_2+x_3+x_4=0$, $x_1+x_2+x_3-x_4=4$, $x_1+x_2-x_3+x_4=-4$, $x_1-x_2+x_3+x_4=2$. [2014-15]

Soln: let $AX = B$ be the sys. of eqns

$$\Rightarrow \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & -1 \\ 1 & 1 & -1 & 1 \\ 1 & -1 & 1 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \\ -4 \\ 2 \end{bmatrix}$$

$X = A^{-1}B$

where, $A = IA$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & -1 \\ 1 & 1 & -1 & 1 \\ 1 & -1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} A$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & -2 \\ 0 & 0 & -2 & 0 \\ 0 & -2 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ -1 & 0 & 0 & 1 \end{bmatrix} A \quad \begin{aligned} R_2 &\to R_2 - R_1 \\ R_3 &\to R_3 - R_1 \\ R_4 &\to R_4 - R_1 \end{aligned}$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1/2 & -1/2 & 0 & 0 \\ 1/2 & 0 & -1/2 & 0 \\ 1/2 & 0 & 0 & -1/2 \end{bmatrix} A \quad \begin{aligned} R_2 &\to -\tfrac{1}{2}R_2 \\ R_3 &\to -\tfrac{1}{2}R_3 \\ R_4 &\to -\tfrac{1}{2}R_4 \end{aligned}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} -1/2 & 1/2 & 1/2 & 1/2 \\ 1/2 & -1/2 & 0 & 0 \\ 1/2 & 0 & -1/2 & 0 \\ 1/2 & 0 & 0 & -1/2 \end{bmatrix} A \quad R_1 \to R_1 - (R_2+R_3+R_4)$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -1/2 & 1/2 & 1/2 & 1/2 \\ 1/2 & 0 & 0 & -1/2 \\ 1/2 & 0 & -1/2 & 0 \\ 1/2 & -1/2 & 0 & 0 \end{bmatrix} A \quad R_2 \leftrightarrow R_4$$

$I = A^{-1}A$

$$X = \begin{bmatrix} -1/2 & 1/2 & 1/2 & 1/2 \\ 1/2 & 0 & 0 & -1/2 \\ 1/2 & 0 & -1/2 & 0 \\ 1/2 & -1/2 & 0 & 0 \end{bmatrix}\begin{bmatrix} 0 \\ 4 \\ -4 \\ 2 \end{bmatrix}$$

$$X = \begin{bmatrix} 1 \\ -1 \\ 2 \\ -2 \end{bmatrix}$$

Thus $\boxed{x_1 = 1, \; x_2 = -1, \; x_3 = 2, \; x_4 = -2}$

Ques 29:- Investigate for what values of λ and μ, the sys. of eqns $x+y+z=6$, $x+2y+3z=10$ and $x+2y+\lambda z=\mu$, has: i) no soln. ii) Unique soln. and iii) infinite no. of solns. [2017-18]

Soln:- Refer Ques 26. (same as that)

Ques 30:- For what values of λ and μ, the sys. of linear eqns: $x+y+z=6$, $x+2y+5z=10$ and $2x+3y+\lambda z=\mu$, has: i) a unique soln., ii) No soln. and iii) Infinite no. of solns. Also find the soln. for $\lambda=2$ and $\mu=8$. [2017-18]

Soln:- Let $AX=B$ be the sys. of eqns

$$\Rightarrow C = [A:B] = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 1 & 2 & 5 & 10 \\ 2 & 3 & \lambda & \mu \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & 4 & 4 \\ 0 & 1 & \lambda-2 & \mu-12 \end{array}\right] \quad \begin{aligned} R_2 &\to R_2 - R_1 \\ R_3 &\to R_3 - 2R_1 \end{aligned}$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & 4 & 4 \\ 0 & 0 & \lambda-6 & \mu-16 \end{array}\right] \quad R_3 \to R_3 - R_2 \quad (7)$$

i) a unique soln:-
If $R(A) = R(C) = 3$ then
$\lambda - 6 \neq 0 \Rightarrow \lambda \neq 6$ and $\mu - 16 \neq 0$
$\Rightarrow \mu \neq 16$ or μ may have any value

ii) No soln:- If $R(A) \neq R(C) \Rightarrow R(A) = 2$ and $R(C) = 3$
$\Rightarrow \lambda - 6 = 0$ and $\mu - 16 \neq 0$
$\Rightarrow \lambda = 6$ and $\mu \neq 16$

iii) Infinite no. of solns:-
If $R(A) = R(C) = 2$
then $\lambda - 6 = 0$ and $\mu - 16 = 0$
$\Rightarrow \lambda = 6$ and $\mu = 16$

iv) Substituting $\lambda = 2$ and $\mu = 8$ in eq (7) we get

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & 4 & 4 \\ 0 & 0 & -4 & -8 \end{array}\right]$$

$$\sim \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 4 \\ 0 & 0 & -4 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 4 \\ -8 \end{bmatrix}$$

$x+y+z=6$ (2)
$y+4z=4$ (3)
$-4z=-8$ (4) $\Rightarrow \boxed{z=2}$

from (3) $y+8=4 \Rightarrow \boxed{y=-4}$
from (2) $x-4+2=6$
$\Rightarrow \boxed{x=8}$

Hence $\boxed{x=8}$, $\boxed{y=-4}$ and $\boxed{z=2}$ is the req. soln.

Eigen Values and Eigen Vectors

1. **Characteristic eq^n:-** The eq^n $\boxed{|A-\lambda I|=0}$ is called the characteristic eq^n of the matrix A (square matrix).

eg. $\boxed{\lambda^3 - 7\lambda^2 + 11\lambda - 5 = 0}$

2. **Characteristic roots or Eigen values:-** The eq^n $\boxed{|A-\lambda I|=0}$ when solved gives the characteristic roots of matrix A.

eg. $\lambda^3 - 7\lambda^2 + 11\lambda - 5 = 0$

$\Rightarrow (\lambda-1)(\lambda-1)(\lambda-5) = 0 \quad \therefore \boxed{\lambda = 1, 1, 5}$

Thus, the characteristic roots of A are $\boxed{1, 1, 5}$

Properties of eigenvalues / Characteristic roots / Latent roots.

1. Any square matrix A and its transpose A' have the same eigen values.
2. The sum of the eigenvalues of a matrix is equal to the trace of the matrix.
3. The product of the eigen values of a matrix A is equal to the determinant of A.
4. If $\lambda_1, \lambda_2, \dots \lambda_n$ are the eigen values of A, then the eigen values of

i) KA are $K\lambda_1, K\lambda_2 \dots K\lambda_n$ ii) A^m are $\lambda_1^m, \lambda_2^m \dots \lambda_n^m$

iii) A^{-1} are $\frac{1}{\lambda_1}, \frac{1}{\lambda_2}, \dots \frac{1}{\lambda_n}$

Note:- The sum of the elements of the principal diagonal of a matrix is called the trace of a matrix.

Characteristic vectors or Eigen Vectors

A column vector X is transformed into column vector Y by means of a square matrix A.

Now, we want to multiply the column vector X by a scalar quantity λ so that we can find the same transformed column vector Y. i.e. $\boxed{AX = \lambda X}$ X is known as eigen vector.

Properties of Eigenvectors:-

1. The eigen vector X of a matrix A is not unique.
2. If $\lambda_1, \lambda_2, \dots \lambda_n$ be distinct eigen values of an $n \times n$ matrix then the corresponding eigen vectors $X_1, X_2, \dots X_n$ form a linearly independent set.
3. If two or more eigen values are equal it may or may not be possible to get linearly independent eigen vectors corresponding to the equal roots.
4. Two eigen vectors X_1 and X_2 are called orthogonal vectors if $\boxed{X_1' X_2 = 0.}$

Ques 31:- If the eigen values of the matrix A are 1, 1, 1 then find the eigen values of $A^2+2A+3I$. [2018-19]

Soln:- Since the eigenvalues of A are 1, 1, 1 then by the property of eigen values.

Eigen values of A^2 are $1^2, 1^2, 1^2 = 1, 1, 1$

Eigen values of $2A$ are $2\cdot1, 2\cdot1, 2\cdot1 = 2, 2, 2$

Eigen values of $3I$ are $3\cdot1, 3\cdot1, 3\cdot1 = 3, 3, 3$

$\therefore$ Eigenvalues of $A^2+2A+3I$ are

$(1+2+3), (1+2+3), (1+2+3) = \boxed{6, 6, 6}$

Similarity Transformation:-

Let A and B be two square matrices of order n. Then B is said to be similar to A if there exists a non-singular matrix P s.t.

$$\boxed{B = P^{-1}AP} \quad (4)$$

eq (4) is called a similar transformation.

Diagonalisation of a Matrix:-

Diagonalisation of a matrix A is the process of reduction of A to a diagonal form 'D'. If A is related to D by a similarity transformation s.t. $\boxed{D = P^{-1}AP}$ then A is reduced to the diagonal matrix D through modal matrix P. D is also called the spectral matrix of A.

Reduction of a matrix to a Diagonal form:-

If a square matrix A of order n has n linearly independent eigen vectors, then the matrix B can be found s.t. $B^{-1}AB$ is a diagonal matrix.

Note:- The matrix B which diagonalises A is called the modal matrix of A and is obtained by grouping the eigenvectors of A into a square matrix.

Ques 32:- Show that the matrix $\begin{bmatrix} 3 & 1 & -1 \\ -2 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}$ is Diagonalizable [2011-12]

Soln:- Let $A = \begin{bmatrix} 3 & 1 & -1 \\ -2 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}$

Characteristic eqn is given by

$|A - \lambda I| = 0$

$\Rightarrow \lambda^3 - \lambda^2(\text{tr. of } A) + \lambda(A_{11} + A_{22} + A_{33}) - |A| = 0$

$\lambda^3 - 6\lambda^2 + 11\lambda - 6 = 0$

$\Rightarrow \lambda = 3, 1, 2$

Now eigenvector corresponding to $\lambda = 3$

$[A - \lambda I]X = 0 \Rightarrow [A - 3I]X = 0$

$$\begin{bmatrix} 3-\lambda & 1 & -1 \\ -2 & 1-\lambda & 2 \\ 0 & 1 & 2-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 & -1 \\ -2 & -2 & 2 \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$x_2 - x_3 = 0 \quad (1)$

$-2x_1 - 2x_2 + 2x_3 = 0 \quad (2)$

$x_2 - x_3 = 0 \quad (3)$

from (1) and (2)

$\left.\begin{array}{l} x_2 - x_3 = 0 \\ -2x_1 - 2x_2 + 2x_3 = 0 \end{array}\right\} \Rightarrow \boxed{x_2 = x_3}$ and $x_1 + x_2 - x_3 = 0$

Let $x_3 = k_1 \Rightarrow x_2 = k_1$ and $x_1 + k_1 - k_1 = 0$

$\boxed{x_1 = 0}$ i.e. $X_1 = \begin{bmatrix} 0 \\ k_1 \\ k_1 \end{bmatrix}$ or $\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$

Eigenvector corresponding to $\lambda = 1$ is given by $[A - I]X = 0$

$$\begin{bmatrix} 2 & 1 & -1 \\ -2 & 0 & 2 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$2x_1 + x_2 - x_3 = 0 \quad (4)$

$-2x_1 + 2x_3 = 0 \quad (5)$

$x_2 + x_3 = 0 \quad (6)$

Solving (4) & (5) by cross-multiplication

$\frac{x_1}{2} = \frac{x_2}{-2} = \frac{x_3}{2} = 1$

$\boxed{x_1 = 2}$, $\boxed{x_2 = -2}$, $\boxed{x_3 = 2}$

$X_2 = \begin{bmatrix} 2 \\ -2 \\ 2 \end{bmatrix}$ or $\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$

Eigenvector corresponding to $\lambda = 2$

$[A - 2I]X = 0$

$$\begin{bmatrix} 1 & 1 & -1 \\ -2 & -1 & 2 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$x_1 + x_2 - x_3 = 0 \quad (7)$

$-2x_1 - x_2 + 2x_3 = 0 \quad (8)$

$x_2 = 0 \quad (9)$

Solving (7) & (8) by cross-multiplication

$\frac{x_1}{1} = \frac{x_2}{0} = \frac{x_3}{1} \Rightarrow \boxed{x_1 = 1}$, $\boxed{x_2 = 0}$, $\boxed{x_3 = 1}$

$X_3 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

Modal Matrix $= \begin{bmatrix} 0 & 1 & 1 \\ 1 & -1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$

Now $\boxed{|M| = 1 \neq 0}$

Hence, eigenvectors are linearly independent.

$\therefore$ the given matrix is diagonalizable.

Ques 33:- If $\alpha_1, \alpha_2, \alpha_3, \ldots \alpha_n$ are the characteristic roots of the n-square matrix A and K is a scalar, P.T the characteristic roots of $[A-KI]$ are $\alpha_1-K, \alpha_2-K, \alpha_3-K, \ldots \alpha_n-K$. [2012-13]

Soln:- Since $\alpha_1, \alpha_2, \ldots \alpha_n$ are the characteristic roots of A, $\therefore$ the characteristic polynomial of A is

$|A-\lambda I| = (\alpha_1-\lambda)(\alpha_2-\lambda)\ldots(\alpha_n-\lambda)$. (1)

The characteristic polynomial of $A-KI$ is

$|A-KI-\lambda I| = |A-(K+\lambda)I|$

$= \{\alpha_1-(\lambda+K)\}, \{\alpha_2-(\lambda+K)\}, \ldots \{\alpha_n-(\lambda+K)\}$ from (1)

$= \{(\alpha_1-K)-\lambda\}, \{(\alpha_2-K)-\lambda\} \ldots \{(\alpha_n-K)-\lambda\}$ which shows that the characteristic roots of $A-KI$ are $\alpha_1-K, \alpha_2-K, \ldots \alpha_n-K$.

Ques 34:- Diagonalize the following matrix A: $\begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$ [2012-13]

Soln:- Given $A = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$

$\Rightarrow |A-\lambda I| = 0 \Rightarrow \begin{vmatrix} 3-\lambda & -1 & 1 \\ -1 & 5-\lambda & -1 \\ 1 & -1 & 3-\lambda \end{vmatrix} = 0$

$\Rightarrow \lambda^3 - \lambda^2(\text{tr. of } A) + \lambda(A_{11}+A_{22}+A_{33}) - |A| = 0$

$\Rightarrow \lambda^3 - 11\lambda^2 + 36\lambda - 36 = 0$

$\Rightarrow \boxed{\lambda = 6, 3, 2}$

eigenvector corresponding to $\lambda = 6$

$[A-6I]X = 0$

$\Rightarrow \begin{bmatrix} -3 & -1 & 1 \\ -1 & -1 & -1 \\ 1 & -1 & -3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$-3x_1 - x_2 + x_3 = 0$
$-x_1 - x_2 - x_3 = 0$
$x_1 - x_2 - 3x_3 = 0$

On solving $X_1 = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$

eigenvector corresponding to $\lambda = 3$

$[A-3I]X = 0$

$\begin{bmatrix} 0 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow$ $-x_2 + x_3 = 0$, $-x_1 + 2x_2 - x_3 = 0$, $x_1 - x_2 = 0$ On solving

$X_2 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

eigen vector corresponding to $\lambda = 2$

$[A-2I]X = 0$

$\begin{bmatrix} 1 & -1 & 1 \\ -1 & 3 & -1 \\ 1 & -1 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$x_1 - x_2 + x_3 = 0$
$-x_1 + 3x_2 - x_3 = 0$
$x_1 - x_2 + x_3 = 0$

On solving $X_3 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$

$M = \begin{bmatrix} 1 & 1 & -1 \\ -2 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$ Now $M^{-1} = \begin{bmatrix} 1/6 & -1/3 & 1/6 \\ 1/3 & 1/3 & 1/3 \\ -\frac{1}{2} & 0 & 1/2 \end{bmatrix}$

$D = M^{-1}AM$

$= \begin{bmatrix} 1/6 & -1/3 & 1/6 \\ 1/3 & 1/3 & 1/3 \\ -1/2 & 0 & 1/2 \end{bmatrix}\begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}\begin{bmatrix} 1 & 1 & -1 \\ -2 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1/6 & -1/3 & 1/6 \\ 1/3 & 1/3 & 1/3 \\ -1/2 & 0 & 1/2 \end{bmatrix}\begin{bmatrix} 6 & 3 & -2 \\ -12 & 3 & 0 \\ 6 & 3 & 2 \end{bmatrix}$

$= \begin{bmatrix} 6 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ Ans.

Ques 35:- Diagonalize the unitary matrix $A = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & i \\ -i & -1 \end{bmatrix}$ [2013-14]

Soln:- Let $A = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & i \\ -i & -1 \end{bmatrix}$

$|A-\lambda I| = 0 \Rightarrow \begin{vmatrix} \frac{1}{\sqrt{2}}-\lambda & \frac{i}{\sqrt{2}} \\ -\frac{i}{\sqrt{2}} & -\frac{1}{\sqrt{2}}-\lambda \end{vmatrix} = 0$

$\Rightarrow \lambda^2 - 1 = 0 \Rightarrow \boxed{\lambda = \pm 1}$

eigenvector corresponding to $\lambda = -1$

$[A+I]X = 0 \Rightarrow \begin{bmatrix} \frac{1}{\sqrt{2}}+1 & \frac{i}{\sqrt{2}} \\ -\frac{i}{\sqrt{2}} & -\frac{1}{\sqrt{2}}+1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$X_1 = \begin{bmatrix} -\sqrt{2}i+i \\ 1 \end{bmatrix}$

eigenvector corresponding to $\lambda = +1$

$\Rightarrow [A-I]X = 0 \Rightarrow \begin{bmatrix} \frac{1}{\sqrt{2}}-1 & \frac{i}{\sqrt{2}} \\ -\frac{i}{\sqrt{2}} & -\frac{1}{\sqrt{2}}-1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$X_2 = \begin{bmatrix} i(1+\sqrt{2}) \\ 1 \end{bmatrix}$

$P = \begin{bmatrix} -\sqrt{2}i+i & i(1+\sqrt{2}) \\ 1 & 1 \end{bmatrix}$

where P represents the modal matrix

$P^{-1} = \frac{\text{adj } P}{|P|} = \frac{-1}{2\sqrt{2}i}\begin{bmatrix} 1 & -i(1+\sqrt{2}) \\ -1 & -\sqrt{2}i+i \end{bmatrix}$

$= \frac{1}{2\sqrt{2}}i\begin{bmatrix} 1 & -i(1+\sqrt{2}) \\ -1 & -\sqrt{2}i+i \end{bmatrix}$

$D = P^{-1}AP$

$= \frac{1}{2\sqrt{2}}i\begin{bmatrix} 1 & -i(1+\sqrt{2}) \\ -1 & -\sqrt{2}i+i \end{bmatrix}\begin{bmatrix} 1 & i \\ -i & -1 \end{bmatrix}\frac{1}{\sqrt{2}}\begin{bmatrix} -\sqrt{2}i+i & i(1+\sqrt{2}) \\ 1 & 1 \end{bmatrix}$

$\boxed{D = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}}$

Ques 36:- If $A = \begin{bmatrix} -1 & 0 & 0 \\ 2 & -3 & 0 \\ 1 & 4 & -2 \end{bmatrix}$, find the eigenvalues of A^2. [2015-16]

Soln:- Given $A = \begin{bmatrix} -1 & 0 & 0 \\ 2 & -3 & 0 \\ 1 & 4 & -2 \end{bmatrix}$.

$|A-\lambda I| = 0$

$\Rightarrow \begin{vmatrix} -1-\lambda & 0 & 0 \\ 2 & -3-\lambda & 0 \\ 1 & 4 & -2-\lambda \end{vmatrix} = 0$

$\Rightarrow (-1-\lambda)[(-3-\lambda)(-2-\lambda)-0] = 0$

$\Rightarrow \boxed{\lambda = -1, -2, -3}$

Now, by using the property of eigenvalues.

If eigenvalues of A are $-1, -2, -3$

$\Rightarrow A^2$ are $(-1)^2, (-2)^2, (-3)^2$

$\boxed{A^2 = 1, 4, 9}$

Ques37:- For what value of 'x' the eigenvalues of the given matrix A are real, $A=\begin{bmatrix} 10 & 5+i & 4 \\ x & 20 & 2 \\ 4 & 2 & -10 \end{bmatrix}$ [2016-17]

Soln:- We know that, eigenvalues of a Hermitian matrix are always real and the above matrix A will be Hermitian iff $\boxed{A^{\theta}=A}$ and that will be possible iff $\boxed{x=5-i}$

Ques38:- Reduce the matrix $P=\begin{bmatrix} 1 & 2 & -2 \\ 1 & 2 & 1 \\ -1 & -1 & 0 \end{bmatrix}$ to diagonal form. [2016-17]

Soln:- $|P-\lambda I|=0$

$$\Rightarrow \begin{vmatrix} 1-\lambda & 2 & -2 \\ 1 & 2-\lambda & 1 \\ -1 & -1 & 0-\lambda \end{vmatrix}=0$$

$\Rightarrow (1-\lambda)(\lambda^2-2\lambda-3)=0$

$\Rightarrow \boxed{\lambda=1,-1,3}$

Eigen vector corresponding to $\lambda=3$

$[P-3I]X=0$

$$\begin{bmatrix} -2 & 2 & -2 \\ 1 & -1 & 1 \\ -1 & -1 & -3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$-2x_1+2x_2-2x_3=0$ (1)

$x_1-x_2+x_3=0$ (2)

$-x_1-x_2-3x_3=0$ (3)

from (2) & (3)

$X_1=\begin{bmatrix} -2 \\ -1 \\ 1 \end{bmatrix}$

eigenvector corresponding to $\lambda=1$

$[A-I]X=0$

$$\begin{bmatrix} 0 & 2 & -2 \\ 1 & 1 & 1 \\ -1 & -1 & -1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$2x_2-2x_3=0$

$x_1+x_2+x_3=0$

$-x_1-x_2-x_3=0$

on solving $X_2=\begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}$

eigenvector corresponding to $\lambda=-1$

$[A+I]X=0$

$$\begin{bmatrix} 2 & 2 & -2 \\ 1 & 3 & 1 \\ -1 & -1 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$2x_1+2x_2-2x_3=0$

$x_1+3x_2+x_3=0$

$-x_1-x_2+x_3=0$

on solving

$X_3=\begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$

$M=\begin{bmatrix} -2 & -2 & 2 \\ -1 & 1 & -1 \\ 1 & 1 & 1 \end{bmatrix}$

$M^{-1}=\begin{bmatrix} -1/4 & -1/2 & 0 \\ 0 & 1/2 & 1/2 \\ 1/4 & 0 & 1/2 \end{bmatrix}$

$\boxed{D=M^{-1}AM}$

$$=\begin{bmatrix} -1/4 & -1/2 & 0 \\ 0 & 1/2 & 1/2 \\ 1/4 & 0 & 1/2 \end{bmatrix}\begin{bmatrix} 1 & 2 & -2 \\ 1 & 2 & 1 \\ -1 & -1 & 0 \end{bmatrix}\begin{bmatrix} -2 & -2 & 2 \\ -1 & 1 & -1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$=\begin{bmatrix} 3 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$ Ans.

Ques39:- Find the eigenvalues of the matrix $\begin{bmatrix} 4 & 2 \\ 2 & 4 \end{bmatrix}$ corresponding to the eigenvector $\begin{bmatrix} 101 \\ 101 \end{bmatrix}$ [2016-17]

Soln:- We know that, $AX=\lambda X$ where $A=\begin{bmatrix} 4 & 2 \\ 2 & 4 \end{bmatrix}$, $X=\begin{bmatrix} 101 \\ 101 \end{bmatrix}$ and λ is the eigen value matrix.

$$\begin{bmatrix} 4 & 2 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} 101 \\ 101 \end{bmatrix}=\begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}\begin{bmatrix} 101 \\ 101 \end{bmatrix}\Rightarrow\begin{bmatrix} 4\times101+2\times101 \\ 2\times101+4\times101 \end{bmatrix}=\begin{bmatrix} 101\lambda_1 \\ 101\lambda_2 \end{bmatrix}$$

Imp.

$\Rightarrow 101\times6=101\lambda_1$, $101\times6=101\lambda_2$ $\Rightarrow \boxed{\lambda_1=6,\ \lambda_2=6}$ are the corresponding eigenvalues.

Ques40:- Find the eigenvalues and the corresponding eigenvectors of the following matrix $A=\begin{bmatrix} 2 & 0 & 1 \\ 0 & 3 & 0 \\ 1 & 0 & 2 \end{bmatrix}$ [2020-21]

Soln:- Given $A=\begin{bmatrix} 2 & 0 & 1 \\ 0 & 3 & 0 \\ 1 & 0 & 2 \end{bmatrix}$

$$|A-\lambda I|=0 \Rightarrow \begin{vmatrix} 2-\lambda & 0 & 1 \\ 0 & 3-\lambda & 0 \\ 1 & 0 & 2-\lambda \end{vmatrix}=0$$

expanding along second row

$(3-\lambda)[(2-\lambda)(2-\lambda)-1]=0$

$(3-\lambda)[4+\lambda^2-4\lambda-1]=0$

$(3-\lambda)[\lambda^2-4\lambda+3]=0$

$(3-\lambda)[(\lambda-3)(\lambda-1)]=0$

$\boxed{\lambda=1,3,3}$

eigenvector for $\lambda=1$

$[A-\lambda I]X=0 \Rightarrow [A-I]X=0$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$x_1+0x_2+x_3=0$ (1)

$0x_1+2x_2+0x_3=0$ (2)

$x_1+0x_2+x_3=0$ (3)

from eq (1) and (2)

$\frac{x_1}{-2}=\frac{-x_2}{0}=\frac{x_3}{2}=1$

$\boxed{x_1=-2}$ $\boxed{x_2=0}$ $\boxed{x_3=2}$ $X_1=\begin{bmatrix} -2 \\ 0 \\ 2 \end{bmatrix}$

eigen vector for $\lambda=3$

$[A-3I]X=0$

$$\Rightarrow\begin{bmatrix} -1 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & -1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$-x_1+0x_2+x_3=0$ (1)

$x_1+0x_2-x_3=0$ (2)

let $\boxed{x_1=k}\Rightarrow\boxed{x_1=x_3}$ from (1) & (2)

$X_2=\begin{bmatrix} k \\ k_1 \\ k \end{bmatrix}$ let $k=1$, $k_1=-1$

$X_2=\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$

and again for $\lambda=3$ (repeated root)

let $k=-1$, $k_1=1$

$X_3=\begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix}$ Ans.

Cayley - Hamilton Theorem (CHT)

Statement:- every square matrix satisfies its own characteristic equation.

Note:- CHT gives another method for computing the inverse of a square matrix. Since the method expresses the inverse of a matrix of order n in terms of $(n-1)$ powers of A, it is the most suitable for computing inverses of large matrices.

Ques 41:- The matrix $A = \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$ satisfies the matrix eqn $A^3-6A^2+11A-I=0$, where I is the identity matrix of order 3. Find A^{-1} using eq (2) [2011-12]

Soln:- The given matrix $A = \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$ satisfies the eqn

$$A^3-6A^2+11A-I=0 \quad (1)$$

To find A^{-1}, pre-multiplying A^{-1} in eq (1) on both sides we get,

$$A^{-1}(A^3-6A^2+11A-I)=0$$

$$A^2-6A+11I-A^{-1}=0$$

$$\Rightarrow \boxed{A^{-1}=A^2-6A+11I} \quad (2)$$

Now $A^2 = A \times A$

$$= \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}\begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 4+0+0 & 0+0-1 & -2+0-3 \\ 10+5+0 & 0+1+0 & -5+0+0 \\ 0+5+0 & 0+1+3 & 0+0+9 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & -1 & -5 \\ 15 & 1 & -5 \\ 5 & 4 & 9 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 4 & -1 & -5 \\ 15 & 1 & -5 \\ 5 & 4 & 9 \end{bmatrix} - 6\begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} + 11\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & -1 & -5 \\ 15 & 1 & -5 \\ 5 & 4 & 9 \end{bmatrix} - \begin{bmatrix} 12 & 0 & -6 \\ 30 & 6 & 0 \\ 0 & 6 & 18 \end{bmatrix} + \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix}$$

$$\Rightarrow A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix} \text{ Ans.}$$

Ques 42:- Find the characteristic eqn of the matrix: $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and hence find the matrix represented by $A^8-5A^7+7A^6-3A^5+A^4-5A^3+8A^2-2A+I=0$, where I is the identity matrix. [2012-13]

Soln:- Given, $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$|A-\lambda I| = \begin{vmatrix} 2-\lambda & 1 & 1 \\ 0 & 1-\lambda & 0 \\ 0 & 0 & 1-\lambda \end{vmatrix} = 0$$

$$\lambda^3 - \lambda^2(\text{tr. of } A) + \lambda(A_{11}+A_{22}+A_{33}) - |A| = 0$$

$$\Rightarrow \lambda^3 - 4\lambda^2 + 5\lambda - 2 = 0$$

Hence, the characteristic eqn is given by $A^3-4A^2+5A-2I=0$

Now, $A^8-5A^7+7A^6-3A^5+A^4-5A^3+8A^2-2A+I$

$$\Rightarrow A^5(A^3-4A^2+5A-2I) - A^4(A^3-4A^2+5A-2I) - 2A^3(A^3-4A^2+5A-2I) - 4A^2(A^3-4A^2+5A-2I) - 7A(A^3-4A^2+5A-2I) - 17(A^3-4A^2+5A-2I) - 33A^2+69A-33I$$

using CHT i.e. $A^3-4A^2+5A-2I=0$

$$\Rightarrow A^5\cdot 0 - A^4\cdot 0 - 2A^3\cdot 0 - 4A^2\cdot 0 - 7A\cdot 0 - 17\cdot 0 - 33A^2+69A-33I$$

$$\Rightarrow -33A^2+69A-33I$$

$$-33\begin{bmatrix} 4 & 3 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + 69\begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - 33\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -132 & -99 & -99 \\ 0 & -33 & 0 \\ 0 & 0 & -33 \end{bmatrix} + \begin{bmatrix} 138 & 69 & 69 \\ 0 & 69 & 0 \\ 0 & 0 & 69 \end{bmatrix} - \begin{bmatrix} 33 & 0 & 0 \\ 0 & 33 & 0 \\ 0 & 0 & 33 \end{bmatrix}$$

$$= \begin{bmatrix} -27 & -30 & -30 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix} \text{ Ans.}$$

Ques 43:- If $A=\begin{bmatrix}1&2&3\\2&4&5\\3&5&6\end{bmatrix}$, find the inverse of A using CHT. [2013-14] [2014-15]

Soln:- $|A-\lambda I|=0$

$$\Rightarrow \begin{vmatrix}1-\lambda & 2 & 3\\ 2 & 4-\lambda & 5\\ 3 & 5 & 6-\lambda\end{vmatrix}=0$$

$\Rightarrow \lambda^3-11\lambda^2-4\lambda+1=0$

$\Rightarrow$ Cayley-Hamilton th. is given by

$\boxed{A^3-11A^2-4A+I=0}$ (1)

On pre-multiplication of A^{-1} on both sides, we get

$A^{-1}(A^3-11A^2-4A+I)=0$

$\Rightarrow A^2-11A-4I+A^{-1}=0$

$\Rightarrow A^{-1}=-[A^2-11A-4I]$

$$A^2=A\times A=\begin{bmatrix}1&2&3\\2&4&5\\3&5&6\end{bmatrix}\begin{bmatrix}1&2&3\\2&4&5\\3&5&6\end{bmatrix}$$

$$=\begin{bmatrix}14&25&31\\25&45&56\\31&56&70\end{bmatrix}$$

$A^{-1}=-(A^2-11A-4I)=-A^2+11A+4I$

$$=-\begin{bmatrix}14&25&31\\25&45&56\\31&56&70\end{bmatrix}+\begin{bmatrix}11&22&33\\22&44&55\\33&55&66\end{bmatrix}+\begin{bmatrix}4&0&0\\0&4&0\\0&0&4\end{bmatrix}$$

$$\Rightarrow A^{-1}=\begin{bmatrix}1&-3&2\\-3&3&-1\\2&-1&0\end{bmatrix}$$ Ans

Ques 44:- Find the characteristic eqn of the matrix $A=\begin{bmatrix}2&-1&1\\-1&2&-1\\1&-1&2\end{bmatrix}$ and verify CHT. Also evaluate $A^6-6A^5+9A^4-2A^3-12A^2+23A-9I$. [2015-16]

Soln:- The characteristic eqn is $|A-\lambda I|=0$

$$\Rightarrow \begin{vmatrix}2-\lambda & -1 & 1\\ -1 & 2-\lambda & -1\\ 1 & -1 & 2-\lambda\end{vmatrix}=0$$

$\Rightarrow \lambda^3-6\lambda^2+9\lambda-4=0$

By Cayley-Hamilton th.

$A^3-6A^2+9A-4I=0$

$$A^2=A\times A=\begin{bmatrix}2&-1&1\\-1&2&-1\\1&-1&2\end{bmatrix}\times\begin{bmatrix}2&-1&1\\-1&2&-1\\1&-1&2\end{bmatrix}=\begin{bmatrix}6&-5&5\\-5&6&-5\\5&-5&6\end{bmatrix}$$

$$A^3=A^2\times A=\begin{bmatrix}6&-5&5\\-5&6&-5\\5&-5&6\end{bmatrix}\begin{bmatrix}2&-1&1\\-1&2&-1\\1&-1&2\end{bmatrix}$$

$$=\begin{bmatrix}22&-21&21\\-21&22&-21\\21&-21&22\end{bmatrix}$$

Verification:- $A^3-6A^2+9A-4I$

$$=\begin{bmatrix}22&-21&21\\-21&22&-21\\21&-21&22\end{bmatrix}-6\begin{bmatrix}6&-5&5\\-5&6&-5\\5&-5&6\end{bmatrix}+9\begin{bmatrix}2&-1&1\\-1&2&-1\\1&-1&2\end{bmatrix}-4\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

$$=\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}=0.$$ So, CHT is verified.

Now $A^6-6A^5+9A^4-2A^3-12A^2+23A-9I$

$=A^3(A^3-6A^2+9A-4I)+2(A^3-6A^2+9A-4I)+5A-I$

$$=0+0+5A-I=\boxed{5A-I}=\begin{bmatrix}10&-5&5\\-5&10&-5\\5&-5&10\end{bmatrix}-\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}=\boxed{\begin{bmatrix}9&-5&5\\-5&9&-5\\5&-5&9\end{bmatrix}}$$

Ques 45:- Express $2A^5-3A^4+A^2-4I$ as a linear polynomial in A where $A=\begin{bmatrix}3&1\\-1&2\end{bmatrix}$. [2016-17]

Soln:- Given $A=\begin{bmatrix}3&1\\-1&2\end{bmatrix}$, The characteristic eqn $|A-\lambda I|=0$

$$\Rightarrow \begin{vmatrix}3-\lambda & 1\\ -1 & 2-\lambda\end{vmatrix}=0 \Rightarrow \boxed{A^2-5A+7I=0}$$

$\Rightarrow \boxed{\lambda^2-5\lambda+7=0}$

Now, $2A^5-3A^4+A^2-4I$ can be written as

$2A^3(A^2-5A+7I)+7A^2(A^2-5A+7I)$
$+21A(A^2-5A+7I)+57(A^2-5A+7I)+138A-403I$

Now, using $A^2-5A+7I=0$

$\Rightarrow 2A^3\cdot 0+7A^2\cdot 0+21A\cdot 0+57\cdot 0+138A-403I$

$\Rightarrow \boxed{138A-403I}$ Ans.

Ques 46:- If $A=\begin{bmatrix}-3 & 2\\ -1 & 0\end{bmatrix}$ then evaluate the value of the expression $(A+5I+2A^{-1})$. [2016-17]

Soln:- Given $A=\begin{bmatrix}-3 & 2\\ -1 & 0\end{bmatrix} \Rightarrow |A| = \begin{vmatrix}-3 & 2\\ -1 & 0\end{vmatrix} = 0\times -3+2\times1 = 2$

and adj $A=\begin{bmatrix}0 & -2\\ 1 & -3\end{bmatrix} \Rightarrow A^{-1}=\frac{\text{adj}A}{|A|}=\frac{1}{2}\begin{bmatrix}0 & -2\\ 1 & -3\end{bmatrix}$

Substituting the value of A^{-1}, A and I in the given eq^n, we get

$A+5I+2A^{-1}=\begin{bmatrix}-3 & 2\\ -1 & 0\end{bmatrix}+5\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}+\frac{2}{2}\begin{bmatrix}0 & -2\\ 1 & -3\end{bmatrix}=\begin{bmatrix}2 & 0\\ 0 & 2\end{bmatrix}$

$\Rightarrow \boxed{A+5I+2A^{-1}=2I}$

Ques 47:- Verify Cayley-Hamilton th. for the matrix $A=\begin{bmatrix}2 & -1 & 1\\ -1 & 2 & -1\\ 1 & -1 & 2\end{bmatrix}$ [2017-18]

Soln:- Refer Ques 44. (for the solution).

Ques 48:- Using Cayley-Hamilton th., find the inverse of the matrix $A=\begin{bmatrix}1 & 2 & 3\\ 2 & 4 & 5\\ 3 & 5 & 6\end{bmatrix}$. Also express the polynomial $B=A^8-11A^7-4A^6+A^5+A^4-11A^3-3A^2+2A+I$ as a quadratic polynomial in A and hence find B. [2018-19]

Soln:- Refer Ques 43. for the solution upto A^{-1}.

Now $B=A^8-11A^7-4A^6+A^5+A^4-11A^3-3A^2+2A+I$

$=A^5(A^3-11A^2-4A+I)+A(A^3-11A^2-4A+I)+A^2+A+I$ (using CHT)

$=A^5\cdot 0+A\cdot 0+A^2+A+I \Rightarrow B=A^2+A+I$

Now, $B=\begin{bmatrix}14 & 25 & 31\\ 25 & 45 & 56\\ 31 & 56 & 70\end{bmatrix}+\begin{bmatrix}1 & 2 & 3\\ 2 & 4 & 5\\ 3 & 5 & 6\end{bmatrix}+\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$

$B=\begin{bmatrix}16 & 27 & 34\\ 27 & 50 & 61\\ 34 & 61 & 77\end{bmatrix}$ Ans.

Ques 49:- Verify Cayley-Hamilton th. for the matrix $A=\begin{bmatrix}4 & 0 & 1\\ 0 & 1 & 2\\ 1 & 0 & 1\end{bmatrix}$ and hence find A^{-1}. [2019-20]

Soln:- $|A-\lambda I|=0 \Rightarrow \begin{vmatrix}4-\lambda & 0 & 1\\ 0 & 1-\lambda & 2\\ 1 & 0 & 1-\lambda\end{vmatrix}=0$

$\Rightarrow \lambda^3-(\text{tr. of }A)\lambda^2+(A_{11}+A_{22}+A_{33})\lambda-|A|=0$

$\Rightarrow \lambda^3-6\lambda^2+8\lambda-3=0$

Now, for CHT

$A^2=A\times A=\begin{bmatrix}4 & 0 & 1\\ 0 & 1 & 2\\ 1 & 0 & 1\end{bmatrix}\begin{bmatrix}4 & 0 & 1\\ 0 & 1 & 2\\ 1 & 0 & 1\end{bmatrix}=\begin{bmatrix}16+0+1 & 0+0+0 & 4+0+1\\ 0+0+2 & 0+1+0 & 0+2+2\\ 4+0+1 & 0+0+0 & 1+0+1\end{bmatrix}$

$=\begin{bmatrix}17 & 0 & 5\\ 2 & 1 & 4\\ 5 & 0 & 2\end{bmatrix}$

$A^3=A^2\times A=\begin{bmatrix}17 & 0 & 5\\ 2 & 1 & 4\\ 5 & 0 & 2\end{bmatrix}\begin{bmatrix}4 & 0 & 1\\ 0 & 1 & 2\\ 1 & 0 & 1\end{bmatrix}=\begin{bmatrix}73 & 0 & 22\\ 12 & 1 & 8\\ 22 & 0 & 7\end{bmatrix}$

Verification:- $A^3-6A^2+8A-3I$

$\begin{bmatrix}73 & 0 & 22\\ 12 & 1 & 8\\ 22 & 0 & 7\end{bmatrix}-6\begin{bmatrix}17 & 0 & 5\\ 2 & 1 & 4\\ 5 & 0 & 2\end{bmatrix}+8\begin{bmatrix}4 & 0 & 1\\ 0 & 1 & 2\\ 1 & 0 & 1\end{bmatrix}-3\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$

$=\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}=0 \Rightarrow$ CHT is verified.

for A^{-1}, pre-multiplication of A^{-1} on both sides of CHT

$A^{-1}(A^3-6A^2+8A-3I)=0$

$\Rightarrow A^2-6A+8I-3A^{-1}=0$

$A^{-1}=\frac{1}{3}(A^2-6A+8I)$

$\Rightarrow \frac{1}{3}\left[\begin{bmatrix}17 & 0 & 5\\ 2 & 1 & 4\\ 5 & 0 & 2\end{bmatrix}-6\begin{bmatrix}4 & 0 & 1\\ 0 & 1 & 2\\ 1 & 0 & 1\end{bmatrix}+8\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}\right]=\frac{1}{3}\begin{bmatrix}1 & 0 & -1\\ 2 & -3 & -8\\ -1 & 0 & 4\end{bmatrix}$ Ans.

10 Year's

University Previous Questions

(Questions Bank)

10 Years AKTU University Examination Questions		Unit-1	
S. No	Questions	Session	Lecture No
1	If $N = \begin{bmatrix} 0 & 1+2i \\ -1+2i & 0 \end{bmatrix}$ is a matrix, then show that $(I-N)(I+N)^{-1}$ is unitary matrix, where I is the identity matrix.	2012-13 (long)	L1
2	If $A = \begin{bmatrix} 2 & 3+2i & -4 \\ 3-2i & 5 & 6i \\ -4 & -6i & 3 \end{bmatrix}$ then show that A is Hermitian and iA is skew-Hermitian.	2013-14 (short)	L1
3	Show that $A = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{bmatrix}$ is a unitary matrix, where ω is the complex cube root of unity.	2016-17 (long)	L1
4	Show that the matrix $\begin{bmatrix} \alpha+i\gamma & -\beta+i\delta \\ \beta+i\delta & \alpha-i\gamma \end{bmatrix}$ is unitary if $\alpha^2+\beta^2+\gamma^2+\delta^2 = 1$	2017-18 (short)	L1
5	Prove that the matrix $\frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1+i \\ 1-i & -1 \end{bmatrix}$ is unitary	2020-21 (short)	L1
6	If $A = \begin{bmatrix} 1 & 2 & 1 \\ \alpha & 0 & 4 \\ 1 & 1 & 1 \end{bmatrix}$ and adj (adj A) = A, Find a.	2011-12 (Long)	L2
7	Explain the working rule to find the inverse of a matrix A by elementary row or column transformations.	2012-13 (short)	L-2
8	For the given matrix $A = \begin{bmatrix} -5 & -3 \\ 2 & 0 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ prove that A^3 = 19 A + 30 I	2016-17 (short)	L2
9	Compute the inverse of the matrix $\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$ by employing elementary row transformations.	2017-18 (long)	L2
10	Find the inverse employing elementary transformation $A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$	2018-19 (long)	L2
11	Find the inverse of the matrix $A = \begin{bmatrix} 2 & 3 & 4 \\ 4 & 3 & 1 \\ 1 & 2 & 4 \end{bmatrix}$	2020-21 (long)	L2
12	Find the value of P for which the matrix $A = \begin{bmatrix} 3 & P & P \\ P & 3 & P \\ P & P & 3 \end{bmatrix}$ is be of rank 1.	2011-12 (Short)	L3
13	Reduce A to echelon form and then to its row canonical form where $A = \begin{bmatrix} 1 & 3 & -1 & 2 \\ 0 & 11 & -5 & 3 \\ 2 & -5 & 3 & 1 \\ 4 & 1 & 1 & 5 \end{bmatrix}$ Hence find the rank of A.	2014-15 (long)	L3

14	Using elementary transformations, find the rank of the following matrix: $A=\begin{bmatrix}2 & -1 & 3 & -1\\ 1 & 2 & -3 & -1\\ 1 & 0 & 1 & 1\\ 0 & 1 & 1 & -1\end{bmatrix}$	2017-18 (long)	L3
15	Find the rank of the matrix $\begin{bmatrix}2 & 2 & 2\\ 2 & 2 & 2\\ 2 & 2 & 2\end{bmatrix}$	2018-19 (short)	L3
16	Find the value of 'b' so that the rank of $A=\begin{bmatrix}2 & 4 & 2\\ 3 & 1 & 2\\ 1 & 0 & b\end{bmatrix}$ is 2.	2019-20 (short)	L3
17	Determine the rank of the matrix: $A=\begin{bmatrix}1 & 1 & 1\\ 0 & 1 & 1\\ 1 & 0 & 1\end{bmatrix}$.	2013-14 (short)	L4
18	Reduce the matrix $\begin{bmatrix}1 & 1 & 1\\ 3 & 1 & 1\end{bmatrix}$ in to the normal form and find its rank.	2017-18 (short)	L4
19	Reduce the matrix A to its normal form when A $=\begin{bmatrix}1 & 2 & -1 & 4\\ 2 & 4 & 3 & 4\\ 1 & 2 & 3 & 4\\ -1 & -2 & 6 & -7\end{bmatrix}$. Hence find the rank of A.	2018-19 (long)	L4
20	Find the rank of the matrix $A=\begin{bmatrix}1 & 3 & 4 & 2\\ 2 & -1 & 3 & 2\\ 3 & -5 & 2 & 2\\ 6 & -3 & 8 & 6\end{bmatrix}$ by reducing it to normal form.	2019 20 (long)	L4
21	State Rank-Nullity theorem	2020-21 (short)	L4
22	Find non-singular matrices P and Q such that PAQ is normal form $\begin{bmatrix}1 & 1 & 2\\ 1 & 2 & 3\\ 0 & 1 & 1\end{bmatrix}$	2020-21 (long)	L4
23	Examine whether the vectors $x_1=[3,1,1]$, $x_2=[2,0,-1]$, $x_3=[4,2,1]$ are linearly independent.	2015-16 (short)	L5
24	Show that the vectors (1, 6, 4), (0, 2, 3) and (0, 1, 2) are linearly independent.	2019-20 (short)	L5
25	Show that the system of equations: $3x+4y+5z=A$, $4x+5y+6z=B$, $5x+6y+7z=C$ are consistent only if A, B and C are in arithmetic progression (A P).	2011-12 (Short)	L6
26	Investigate for what values of λ *and* μ the simultaneous equations $x+y+z=6$, $x+2y+3z=10$, $x+2y+\lambda z=\mu$ have (i) No Solution, (ii) a Unique Solution and (iii) an infinite number of Solutions.	2012-13 2015-16 (Long)	L6
27	Test the consistency and solve the following system of equations. $2x-y+3z=8$, $x+2y+z=4$ and $3x+y-4z=0$.	2013-14 (Short)	L6
28	Solve by calculating the inverse by elementary row operations :	2014 15 (Long)	L6

	$x_1+x_2+x_3+x_4=0$, $x_1+x_2+x_3-x_4=4$, $x_1-x_2+x_3+x_4=2$, $x_1+x_2-x_3+x_4=-4$.		
29	Investigate for what values of λ and μ, the system of equations $x+y+z=6$, $x+2y+3z=10$ and $x+2y+\lambda z=\mu$, has: (i) No solution (ii) Unique solution and (iii) Infinite no. of solutions	2017-18 (long)	L6
30	For what values of λ and μ, the system of linear equations: $x+y+z=6$, $x+2y+5z=10$ and $2x+3y+\lambda z=\mu$, has: (i) a unique solution (ii) no solution and (iii) infinite solution. Also find the solution for $\lambda=2$ and $\mu=8$.	2019-20 (long)	L6
31	If the Eigen values of the matrix A are 1, 1, 1 then find the Eigen values of $A^2+2A+3I$	2018-19 (short)	L8
32	Show that the matrix $\begin{bmatrix}3 & 1 & -1\\ -2 & 1 & 2\\ 0 & 1 & 2\end{bmatrix}$ is Diagonalizable.	2011-12 (short)	L9
33	If $\alpha_1,\alpha_2,\alpha_3,\ldots\ldots\alpha_n$ are the characterstic roots of the n-square matrix A and k is a scalar, prove that the characterstic roots of [A-kI] are $\alpha_1-k,\alpha_2-k,\alpha_3-k,\ldots\ldots\alpha_n-k$.	2012-13 (short)	L9
34	Diagonalize the following matrix A. $A=\begin{bmatrix}3 & -1 & 1\\ -1 & 5 & -1\\ 1 & -1 & 3\end{bmatrix}$.	2012-13 (Long)	L9
35	Diagonalize the unitary matrix $A=\frac{1}{\sqrt{2}}\begin{bmatrix}1 & i\\ -i & -1\end{bmatrix}$.	2013-14 (Long)	L9
36	If $A=\begin{bmatrix}-1 & 0 & 0\\ 2 & -3 & 0\\ 1 & 4 & -2\end{bmatrix}$, find the eigen values of A^2.	2015-16 (short)	L9
37	For what value of 'x', the Eigen values of the given matrix A are real $A=\begin{bmatrix}10 & 5+i & 4\\ x & 20 & 2\\ 4 & 2 & -10\end{bmatrix}$	2016 17 (short)	L9
38	Reduce the matrix $P=\begin{bmatrix}1 & 2 & -2\\ 1 & 2 & 1\\ -1 & -1 & 0\end{bmatrix}$ to diagonal form.	2016-17 (long)	L9
39	Find the Eigen value of the matrix $\begin{bmatrix}4 & 2\\ 2 & 4\end{bmatrix}$ corresponding to the eigen vector $\begin{bmatrix}101\\ 101\end{bmatrix}$.	2016-17 (long)	L9
40	Find the Eigen values and the corresponding Eigen vectors of the following matrix $A=\begin{bmatrix}2 & 0 & 1\\ 0 & 3 & 0\\ 1 & 0 & 2\end{bmatrix}$	2020-21 (long)	L9

41	The matrix A = $\begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$ satisfies the matrix equation $A^3 - 6A^2 + 11A - I = 0$, where I is an identity matrix of order 3. Find A^{-1}.	2011-12 (short)	L10
42	Find the characteristic equation of the matrix: A= $\begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and hence find the matrix represented by $A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I = 0$, where I is the identity matrix.	2012-13 (short)	L10
43	If A = $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$, find the inverse of A using Cayley Hamilton Theorem.	2013-14 2014-15 (short)	L10
44	Find the characteristic equation of the matrix A = $\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ and verify Cayley Hamilton theorem. Also evaluate $A^6 - 6A^5 + 9A^4 - 2A^3 - 12A^2 + 23A - 9I$.	2015-16 (Long)	L10
45	Express $2A^5-3A^4+A^2-4I$ as a linear polynomial in A where A= $\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$	2016-17 (short)	L10
46	If A = $\begin{bmatrix} -3 & 2 \\ -1 & 0 \end{bmatrix}$ then evaluate the value of the expression $(A+5I+2A^{-1})$	2016-17 (short)	L10
47	Verify Cayley-Hamilton theorem for the matrix A = $\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$	2017-18 (long)	L10
48	Using Cayley-Hamilton theorem, find the inverse of the matrix A = $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$. Also express the polynomial B = $A^8 - 11A^7 - 4A^6 + A^5 + A^4 - 11A^3 - 3A^2 + 2A + I$ as a quadratic polynomial in A and hence find B.	2018-19 (long)	L10
49	Verify Cayley-Hamilton theorem for the matrix A = $\begin{bmatrix} 4 & 0 & 1 \\ 0 & 1 & 2 \\ 1 & 0 & 1 \end{bmatrix}$ and hence find A^{-1}.	2019-20 (long)	L10